Optimum design of cold-formed steel columns by using micro genetic algorithms

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Abstract

Cold-formed steel members such as beams and columns have the great flexibility of cross-sectional profiles and sizes available to structural steel designers. However, this flexibility makes the selection of the most economical section difficult for a particular situation. In this study, micro genetic algorithm (MGA) is used to find an optimum cross section of cold-formed steel channel and lipped channel columns under axial compression. Flexural, torsional and torsional–flexural buckling of columns and flat-width-to-thickness ratio of web, flange and lip are considered as constraints. The design curves are generated for optimum values of the thickness, the web flat-depth-to-thickness ratio, the flange flat-width-to-thickness ratio for columns. As numerical results, the optimum design curves are presented for various load level and column lengths.

Keywords: Cold-formed steel channel columns; Optimization; Micro genetic algorithm

1. Introduction

Cold-formed steel members have been widely used in civil engineering structures. Their applications include bridges and buildings. Recently, they are utilized for the various purposes by developing new and various section profiles and verifying the structural behavior through the numerous experimental and theoretical researches. The advantages of cold-formed steels include their ease of fabrication, high strength/weight ratio and suitability for a wide range of applications. Designers can easily vary the profiles of the cross-section according to their structural or constructional needs.

Different from hot-rolled shapes that must have been selected from precast goods, cold-formed steels have permanent choices, thus provide full advantages to designers. However, this merit can be sometimes disadvantage as designers must find the optimum section size and shape for given conditions. For this reason, optimization techniques of cold-formed steel section have been studied since late 1990s. Adeli and Karim [1] investigated the optimum hat section of cold-formed steel beams under uniformly distributed loading. Mackie [2] studied the optimization of cold-formed steel channel, lipped channel and hat section by applying neural network. Recently, Tian and Lu [3] minimized weight of cold-formed steel channel section, either with or without lips under compression using sequential quadratic programming (SQP) algorithm and simply-minded optimization procedure. Schafer [4] studied knowledge-based global optimization of cold-formed steel columns with direct strength method. More recently, Lee et al. [5] used micro genetic algorithms (MGA) to optimize cold-formed channel beams under uniformly distributed loads. They found that simply supported cold-formed steel channel beams under uniformly distributed load are governed by bending strength and web crippling rather than the other structural constraints.

In this paper, the optimization model given in Ref. [5] is extended to cold-formed steel channel and lipped-channel column under axial compression passing through the centroid of the cross section. In optimization process, all the possible structural effects including flexural, torsional...
and torsional–flexural buckling of columns and flat-width-to-thickness ratio of web, flange and lip are considered as constraints, and the optimum design cross section is determined for various levels of loading and column length.

2. Design of cold-formed steel columns

In this section, design of cold-formed steel columns is prepared on the basis of the 2001 edition of the North American Specification for the Design of Cold-Formed Steel Structural Members [6] and the 2002 edition of the AISI Cold-Formed Steel Design Manual [7].

2.1. Effective widths of compression elements

Compressive elements of cold-formed steel generally do not collapse when the buckling stress is reached. Instead, an additional load can be carried by the element after buckling by means of a redistribution of stress and this phenomenon is known as post-buckling strength. Because the post-buckling strength is not treated enough in the structural design criteria of hot-rolled shapes, American Iron and Steel Institute (AISI) [7] estimates the strength using a concept of “effective width” which was first introduced by von Kármán [8] in 1932 to consider post-buckling strength.

2.1.1. Effective widths of stiffened and unstiffened elements

According to AISI [7], the effective widths of stiffened and unstiffened elements as shown in Fig. 1 can be determined by the following equations:

\[
b = \begin{cases} 
  w & (\lambda \leq 0.673), \\
  \rho w & (\lambda > 0.673), 
\end{cases}
\]

where \( w \) is the flat width as shown in Fig. 1

\[
\rho = \left(1 - 0.22/\lambda\right)/\lambda, 
\]

\[
\lambda = \sqrt{\frac{F_n}{F_{cr}}},
\]

In Eq. (3), \( F_{cr} \) is the buckling stress of the plate given by

\[
F_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{1}{w}\right)^2, 
\]

where \( t \) is the thickness of uniformly compressed stiffened and unstiffened elements, \( \mu \) the Poisson’s ratio of steel, and, \( F_n \) the stress in compression element determined in Section 2.2, \( E \) the modulus of elasticity, \( k \) the for plate buckling coefficient, 4 for stiffened elements and 0.43 for unstiffened elements

2.1.2. Effective widths of elements with an edge stiffener

Effective widths of elements with an edge stiffener like flange of lipped channel as shown in Fig. 2 can be determined by the following equations:

For \( w/t \leq 0.328 \)

\[
I_0 = 0 \quad \text{(no edge stiffener needed)}, 
\]

\[
b_1 = b_2 = w/2 \quad \text{(see Fig. 2)}, 
\]

\[
d_s = d'_s \quad \text{for simple lip stiffener}, 
\]

\[
A_s = A'_s \quad \text{for other stiffener shapes}, 
\]

For \( w/t > 0.328 \)

\[
b_1 = b/2(R_1) \quad \text{(see Fig. 2)}, 
\]

\[
b_2 = b - b_1 \quad \text{(see Fig. 2)}, 
\]

\[
d_s = d_s(R_1) \quad \text{for simple lip stiffener}, 
\]

\[
A_s = A_s(R_1) \quad \text{for other stiffener shapes}, 
\]

where

\[
S = 1.28\sqrt{\frac{E}{f}} \quad \text{(for compression members, } f = F_n), 
\]

\[
(R_1) = I_s/I_a \leq 1, 
\]

\[
I_a = 399t\left(\frac{w/t}{S} - 0.328\right)^3 \leq t^4 \left[11.5\frac{w/t}{S} + 5\right], 
\]

\[
d_s = \left(d^3 t \sin^2 \theta\right)/12
\]

\[
D, d = \text{Actual stiffener dimensions}
\]

Fig. 1. Effective width of stiffened and unstiffened elements.

Fig. 2. Elements with simple lip edge stiffener.
the reduced area of stiffener, $A_e$, the effective area of stiffener

$$ n = \left[ 0.582 - \frac{w/t}{4s} \right] \geq \frac{1}{3}, \quad (17) $$

The effective width, $b$, can be calculated in accordance with Section 2.1.1 with $k$ as given in Table 1.

2.2. Nominal axial strength

The nominal axial strength (compressive resistance), $P_n$, of cold-formed steel columns under axial compression passing through the centroid of the effective section can be calculated as follows:

$$ P_n = A_x F_n, \quad (18) $$

where $A_x$ is effective area calculated in compliance with Section 2.1, and nominal compressive stress $F_n$ is determined as follows:

$$ F_n = \begin{cases} 
(0.658 \frac{E}{I_x}) F_y & (\lambda \leq 1.5), \\
(0.877 \frac{E}{I_x}) F_y & (\lambda > 1.5), 
\end{cases} \quad (19) $$

where

$$ \lambda = \frac{F_n}{F_x}, \quad (20) $$

where $F_x$ is the the least of the elastic flexural, torsional and torsional–flexural buckling stress determined according to Sections 2.2.1 and 2.2.4.

2.2.1. Sections not subject to torsional or torsional–flexural buckling

Sections which can be shown not to be subjected to torsional or torsional–flexural buckling, the elastic flexural buckling stress, $F_x$, can be determined as follows:

$$ F_x = \frac{\pi^2 E}{(KL/r)^2}, \quad (21) $$

where $K$ is the effective length factor, $L$ the laterally unbraced length of member, $r$ the radius of gyration of full unreduced cross section about axis of buckling.

2.2.2. Singly-symmetric sections subject to torsional or torsional–flexural buckling

For singly-symmetric sections subject to torsional–flexural buckling, $F_x$ shall be taken as the smaller of $F_e$ calculated according to Section 2.2.1 and $F_c$ calculated as follows:

$$ F_c = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta \sigma_{ex} \sigma_t} \right], \quad (22) $$

where

$$ \beta = 1 - \left( x_0/r_0 \right)^2, \quad (23) $$

$$ \sigma_{ex} = \frac{\pi^2 E}{(K_x L_x/r_x)^2}, \quad (24) $$

$$ \sigma_t = \frac{1}{A r_0^2} \left[ GJ + \frac{\pi^2 E C_w}{(K_x L_x)} \right], \quad (25) $$

where $x_0$ is the distance from shear center to centroid along principal $x$-axis, taken as negative, $r_0$ the polar radius of gyration of cross section about shear center

$$ = \sqrt{r_x^2 + r_y^2 + x_0^2}, \quad (26) $$

$r_x, r_y$ is the radii of gyration of cross section about centroidal principal axes, $K_x, K_t$ the effective length factor for bending about $x$-axes and for twisting, $L_x, L_t$ the unbraced length of member for bending about $x$-axes and for twisting, $A$ the full unreduced cross-sectional area, $G$ the shear modulus, $J$ the Saint-Venant torsional constant of cross section, $C_w$ the torsional warping constant of cross section.

2.3. Dimensional limits of elements

According to 2002 edition of the AISI Cold-Formed Steel Design Manual [7], maximum flat-width-to-thickness ratios, $b/t$, of unstiffened compression elements such as compression flange of channel section or stiffened compression element having one longitudinal edge connected to a web and the other edge is stiffened by a simple lip such as compression flange of lipped channel section is limited to 60, maximum flat-depth-to-thickness ratios, $h/t$, of stiffened compression element with both longitudinal edges connected to other stiffened elements such as compression web of channel and lipped channel section is limited to 300 and maximum flat-width-to-thickness ratios of lip, $d/t$, is limited to 14.

2.4. Limits of slenderness ratios

In accordance with 2002 edition of the AISI Specification [7], the slenderness ratio, $KL/r$, of all compression members should not exceed 200, except that during construction only, $KL/r$ should not exceed 300. Therefore,
maximum slenderness ratio, 200, is applied to the optimization of cold-formed steel column sections in this study.

3. Optimization formulation for cold-formed steel columns

Optimum design formulation of cold-formed steel columns is derived based on the AISI specification [7] mentioned in the previous section. The objective function is cross sectional area of columns consisted of four design variables, depth \(H\), width \(B\), thickness \(t\) and corner radius \(R\), for channel columns. For lipped channel columns, one more design variable, lip length \(p\) is added to design variables of channel columns. Eqs. (27) and (28) represent the objective function of channel and lipped channel columns, and Eq. (29) represents constraints of the present problem (Fig. 3).

(a) Objective function of channel columns
Minimize \(f(x) = (X_1 + 2X_2 - 0.86X_3 - 2.43X_4)X_4\)
\((X_1 = H, \ X_2 = B, \ X_3 = R, \ X_4 = t)\). \hfill (27)

(b) Objective function of lipped channel columns
Minimize \(f(x) = (X_1 + 2X_2 - 2X_3 - 1.72X_4 + 4.86X_5)X_5\)
\((X_1 = H, \ X_2 = B, \ X_3 = D, \ X_4 = R, \ X_5 = t)\). \hfill (28)

Subject to \(P \leq \frac{P_n}{\Omega_c}\),
\(\frac{h}{t} \leq 500\),
\(\frac{b}{t} \leq 60\),
\(\frac{d}{t} \leq 14\),
\(\frac{KL}{r} \leq 200\), \hfill (29)

where \(f(x)\) is cross-sectional area of cold-formed steel channel and lipped channel columns, \(P\) is applied load passing through the centroid of the effective section, and \(h, b, d,\) and \(d\) are flat widths of web, flange and lip, respectively.

Upper and lower bounds of each design variable are considered for manufacturing, and used as side constraints as given in Eqs. (30)–(34). The side constraints are subject to change as manufacturing section size changes
\(H^L \leq H \leq H^U\), \hfill (30)
\(B^L \leq B \leq B^U\), \hfill (31)
\(D^L \leq D \leq D^U\), \hfill (32)
\(R^L \leq R \leq R^U\), \hfill (33)
\(t^L \leq t \leq t^U\). \hfill (34)

Since genetic algorithm (GA) is unconstrained optimization technique, the optimization formulation is transformed by using penalty function method as given in Eq. (35). Penalty terms are added to original objective function when any constraint is violated. The amount of penalty is decided by the ratio of violation. Each gene might have higher fitness value in fitness evaluation in GA as each gene has lower objective function value. The opportunity to be a parent gene is reduced as having lower fitness value, therefore, the gene with lower fitness value would have lower survival probability on the next generation. Those genes would become extinct after several generation and survived genes with higher fitness value should produce better children on the next generation
Minimize \(F(X, M) = f(X)\)
\((\text{Required Strength} \leq \text{Allowable Strength})\),
\(F(X, M) = f(X) + \text{penalty}(X)\)
\((\text{Required Strength} > \text{Allowable Strength})\), \hfill (35)

\(\text{penalty}(X) = r \left\{ \frac{\Omega}{P_n} \frac{KL}{200r} + \frac{1}{500} \frac{h}{t} + \frac{1}{60} \frac{b}{t} + \frac{1}{14} \frac{d}{t} \right\}^2\), \hfill (36)

where \(r\) is a penalty constant and \(\alpha = 1.0\) for lipped channel sections, \(\alpha = 0\) for channel sections.

4. Micro genetic algorithms (MGAs)

4.1. Genetic algorithms (GAs)

GAs are search algorithms based on the mechanics of natural selection and natural genetics. They combine survival of the fittest among string structures with a structured yet randomized information exchange to form a search algorithm with some of the innovative flair of human search. While heuristic search method such as simulated annealing or taboo search uses one solution on
their process to find optimum point, GAs use population of solutions to find optimum point. It is known that mathematical optimization methods which use gradient vector and Hessian are difficult to find optimum point if there are a lot of local optima around optimum point and steep gradient around optimum point [9]. GAs do not use gradient vector and Hessian, but use object function value during their search. In its standard form, application of a GA requires the representation of design variables in terms of bit strings that are counterparts of natural chromosomes, made up of a string of genes.

4.2. Micro genetic algorithms

It is known that more than 30 individuals should be used in GAs in order to prevent the genetic drift [10]. As population size increases, the algorithms find better solution. Bigger population size, however, requires more computational time to find the optimum solution [11]. In this reason, Goldberg [9, 11] proposed serial genetic algorithms (SGAs) which use small population size compared to conventional GAs. Based on SGAs, Krishnakumar [12] proposed MGAs in 1989. MGAs use a relatively smaller population size than SGAs resulting in less computational time. Moreover, MGAs use elitism and convergence checking with re-initialization to obtain the optimal or near optimal solutions.

In order to determine the appropriate population size in this study, the various population sizes of 10, 12, 14, 16 and 18 individuals are tested, respectively. It appears that all the five population sets show similar convergence rate with different computational time. Accordingly, the population size of 10 individuals is selected. The flowchart of MGAs used in this study is illustrated in Fig. 4.

The modified MGA with multi point crossover shows better performance than the algorithms with one point crossover. In the MGAs, there are two stopping criteria called inner and outer criteria. If the solution does not evolve within 10 inner loops, the inner loop is terminated, and the outer loop is performed until the total generation reaches 500 times. Therefore, the number of inner and outer loops varies, but the total number of generation is fixed. Moreover, crossover rate is set as 1.0 therefore all population must perform crossover operation at every generation. There is no use of mutation operation because restart operation and 1.0 crossover rate provide adequate variability.

5. Sensitivity analysis

In this section, sensitivity analysis of design variables to cross sectional area and allowable compressive loads are performed to search governing factor in optimization. Dimensions of section such as flange, web and lip length, thickness and corner radius are changed from 70% to 140% separately based on the standard section. Height ($H$), width ($B$), lip length ($D$), corner radius ($R$), and thickness ($t$) of the standard lipped channel section are set to 20, 5, 2, 0.45, and 0.185, respectively. For standard channel section, lip length ($D$) is zero and all the other dimensions are the same as the lipped channel section.

The results of the sensitivity analysis of cross sectional area for the channel section and the lipped channel section are shown in Figs. 5 and 6. For both cases, the change of cross sectional thickness has the most significant impact on the cross sectional area. The next sensitive variables are shown to be the length in web and flange. The change of the lip length has little impact on the change of cross sectional area. As the radius of the section increases, the total cross sectional area rather decreases. This is because when the width and height of cross section are fixed, the flange and the web length decrease relatively as corner radius increases.

The results of sensitivity analysis for the allowable compressive strength of channel and lipped-channel
column are shown in Figs. 7 and 8. According to the analysis, the changes of flange length have the most significant impact on the allowable compressive strength in both channel and lipped-channel columns. The thickness of the cross section also has some influence on the allowable compressive strength of the columns. As for the case of sensitivity to the cross sectional area, the changes in the lip length and corner radius have little influence on the allowable compressive strength. It should be noted that the compressive strength slightly increases until the web length reaches 90% of standard specification. As the web length increases, however, the allowable compressive strength rather decreases. That is, when all the other variables \((B, D, R, \text{and } t)\) are fixed, the radius of gyration of the cross section decreases as the web length increases. Accordingly, the elastic flexural buckling stress, \(F_e\) in Eq. (21), which decides buckling stress of the channel and the lipped-channel columns, decreases.

6. Design examples

In this parametric study, a simply supported cold-formed steel channel and lipped-channel columns under axial compression \((P)\) passing through the centroid of cross section is considered as shown in Fig. 9. The length of the column is assumed to be varied from 0.5 to 5.0 m and the applied load is assumed from 10 to 30 kN. The yield stress and modulus of elasticity of steel are 240 MPa and 210 GPa, respectively.

As can be seen from the sensitivity analysis, the lip length \((D)\) and the corner radius \((R)\) have little impact on the changes in the cross-sectional area and allowable compressive strength. Accordingly, they are fixed at 2.0 and 0.4 cm, respectively, to save computational time, and the depth of the web \((H)\), the length of the flange \((B)\) and the thickness of the cross section \((t)\) are considered as design variables.

Convergence curves of the cross-sectional area of channel and lipped-channel columns are illustrated with respect to the generation size in Figs. 10 and 11 for \(L = 300\) cm and \(P = 10\) kN. Three different simulations were performed with error less than 3.0%. It shows that the optimized cross sectional area for the lipped channel column is smaller than that of the channel column for the same load level. This is due to the fact that the effective flange width of the lipped channel column is raised by the load carrying capacity of the lip. In general, the lipped channel section is about 15% more effective than the channel section.

6.1. Optimum section for fixed height

In order to investigate the effect of the length and the thickness of the flange on the optimum section, the height
of the cross section and the lip length are fixed at 20 and 2 cm, respectively, and the thickness and the flange length are varied. The optimum thickness and the flange width-to-thickness ratio of the flange for channel and lipped channel sections with respect to the span length are shown in Figs. 12 and 13 for various intensities of the load.

For relatively short span length, thickness is not much sensitive to the change of span length. The optimum section is determined by either the upper limit of flange width \( b_u \) or the limit of flange flat width-to-thickness ratio \( \frac{b}{t} = 60 \). For a channel beam with \( P = 10 \) kN, the thickness and the flange length are determined by flange flat width-to-thickness ratio constraint from 0.5 to 2.5 m of the column length, but as the load and the length of column increase, the allowable compressive strength constraint control the behavior of channel column. For channel section beams with \( P = 30 \) kN, the flange flat width-to-thickness ratio rapidly increases until the flange width reaches its upper limit. After the flange width reaches its upper limit, the increase of thickness is accelerated, and thus, the flange flat width-to-thickness ratio constantly decreases. The slenderness ratio limit controls the sections with longer span length. For lipped channel section, the flange width-to-thickness ratio rather gradually increases depending on the load intensity. This is because the lip provides the wider range of effective area in the flange of the lipped channel section.

6.2. Optimum section for fixed width

In order to investigate the effect of the length and the thickness of the web on the optimum section, the width of the cross section and the lip length are fixed at 5 and 2 cm, respectively, and the thickness and the web length are varied. The optimum thickness and the flat-width-to-thickness ratio of the web for channel and lipped channel sections with respect to the span length are shown in Figs. 14 and 15 for various intensities of the load. The allowable compressive strength constraint controls the behavior of column throughout the whole range when the width of cross section is fixed. For example, the
optimization results of channel column can be explained for the case of \( P = 20 \text{ kN} \) as follows:

**Range A:** The allowable compressive strength of channel column is determined by the flexural–torsional buckling stress in Eqs. (22). While the web length does not change, the thickness increases in order to find the minimum cross section, and the web flat depth-to-thickness decreases linearly. The web length increase does not lead to the increase of the allowable compressive strength of column. Accordingly, the web length remains unchanged, and the thickness increases until the web becomes fully effective.

**Range B:** The allowable compressive strength of column is determined by the flexural–torsional buckling stress just like Range A, but the increase in the thickness is decelerated and the web flat depth-to-thickness ratio increases. This is because the web is now fully effective, and the thickness and the web length increase at the same time.

**Range C:** While the thickness rapidly increases, the web flat width-to-thickness ratio decreases in Range C. That is, due to the increase of the slenderness ratio of the column, the allowable compressive strength of column is determined by flexural buckling stress in Eqs. (21).

**Range D:** The web flat depth-to-thickness ratio increases rapidly again over 270 cm of the column length. The thickness reaches the upper limit, and the web length increases until it reaches the limit of the slenderness ratio. Generally the lipped-channel column shows similar behavior to the channel column, but the optimum thickness is determined by the lip width-to-thickness ratio for relatively short column.

### 7. Conclusion

In this study, the shape optimization of cold-formed steel channel and lipped-channel columns under axial compression was presented by using MGAs. The optimum solution is obtained by using a small population compared to SGAs, and the parameters used in genetic operation can be predetermined.

MGAs showed excellent performance on the minimum weight design of cold-formed steel channel and lipped channel columns. It is found that the lipped-channel section is about 15% more efficient than the channel section for all the applied load level. According to the sensitivity analysis, the thickness and the flange length are found to be sensitive on the cross section and the allowable compressive load. The allowable compressive strength is the most significant constraint that determines the specification of optimum cross section, and in some cases, the flange and the lip flat width-to-thickness ratio, and the slenderness ratio operate become significant constraints. The study for more various sections of cold-formed steel awaits further attention.

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### References


